A NEW UNDERSTANDING FOR CHARACTERIZING ACOUSTIC RECTANGULAR-ROD WAVEGUIDES AND 3D DISCONTINUITIES

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Abstract

Our investigation on the full finite-element analysis has been extended to the precise discussions on the vector-modal behavior on uniform rectangular waveguides, and has found some specific vector-modal behavior in the fundamental mode of the pseudo-longitudinal, flexural and tortional waves. As an example, we show here for the first time that the modal behavior of the fundamental flexural mode on a 3D rectangular waveguide can be discussed well by the behavior of the fundamental Lamb mode on the correspondingly-approximated 2D elastic-plate waveguide. This new understanding is then applied to develop a novel 2D Lamb-mode-waveguide model for analyzing the transmission characteristics of double-step discontinuities (or a resonator cavity) on 3D acoustic rectangular waveguide.

1. INTRODUCTION

In proportion as acoustic devices are highly developed, the ground of design approaches must shift from two-dimensional (2D) standpoint to 3D one, and also from a simple analytical method to a full analytical method. Indeed, there are several interesting numerical models for 3D acoustic waveguides and devices. Although there are several work [1-6] on 3D-waveguide characterization, these approaches are actually complicated and it is difficult to see through the results physically for some of them. As for acoustic-device study [7,8], they assume that such devices are to be periodic infinitely in one or two directions. Such an assumption is justified only when the number of the unit cell is of the order of several hundreds to a few thousands. For such periodic structures, the vector-field behavior on the entire structure can be solved from the knowledge of that only in one cell, introducing the phase-shifting condition that can not be an actual boundary condition for practical 3D structures.

On the other hand, highly developed functional devices are often installed on practical 3D acoustic waveguides with, for example, a rectangular cross section or a circular one, and they will sometimes include only a few unit cells or simple discontinuities like single-step and double-step discontinuities. However, to our knowledge, step-discontinuity problems on, for example, acoustic rectangular waveguides, have not been discussed well by the full-wave analysis. For such investigation, we should first understand the modal behavior of vector field on an uniform 3D acoustic waveguides in full detail. Thus, the central focus of this paper is summarized into the following three items:

(1) To explain our new understanding on the modal-field behavior on 3D rectangular waveguides,
(2) To analyze accurately the transmission characteristics of the double-step discontinuities (or a resonator cavity) on 3D rectangular waveguide by the vector FEM, and
(3) To develop a novel 2D model for analyzing the transmission characteristics for 3D model mentioned in (2) above.

2. MODAL-FIELD BEHAVIOR ON ACOUSTIC 3D RECTANGULAR WAVEGUIDES

Since the modal fields on acoustic rectangular waveguides are very complicated due to a hybrid behavior of both the volume-like and surface-wave-like waves, it is difficult, at the present time, to unify the classification of the modes and their naming. However, it is true that the characteristic modes on an acoustic rectangular waveguide, of which the cross section is shown in Fig. 1, can be classified into the following three groups; the pseudo-longitudinal modes, the flexural modes and the torsional modes. These mode groups are characterized by the boundary conditions on both the yz plane at $x = 0$ ("Y-plane") and the xz plane at $y = 0$ ("X-plane"). As an example, the flexural-mode groups has the Y and X planes as the symmetric plane and the anti-symmetric plane, respectively,

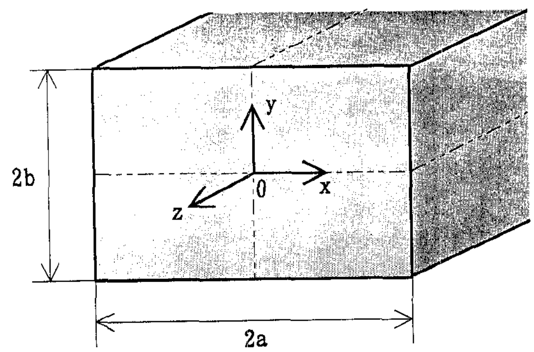

Fig. 1. Cross section of the acoustic rectangular waveguide.

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and vice versa. Then, we first explain our new understanding on the modal-field behavior of, as an example, the fundamental flexural mode on the acoustic rectangular waveguide from the viewpoints of both the dispersion relations and the particle-displacement-vector plots.

In our accurate FEM calculations, an aluminum (the density $\rho = 2.69$ [g/cm^3]) is assumed as the elastic material, and the relative value a/b is varied. Fig. 2 shows the dispersion curves of the normalized frequency $\omega b/\pi v_s$ versus the normalized phase constant $\beta b/\pi$, for different a/b values. This figure shows the dispersion relations in the frequency range, in which only the fundamental mode can propagate, because most of devices will be designed in such a frequency range. On the other hand, the Y plane is now the symmetric one, so that we tend to consider the SH wave as the existing mode when the width a is increased sufficiently. In Fig. 2, the thick-solid curve indicates the dispersion curve for the SH wave when $a \to \infty$, while the thickness b is kept constant. This SH-wave curve is actually a slant-straighten line with some finite angle to the frequency axis, while the curves for the flexural mode rise with the right angle to the frequency axis at the origin. Thus, the modal behavior of the fundamental flexural mode on the rectangular waveguide is never tending to that of the SH wave on the elastic plate with the thickness b ($a \to \infty$), even if the a value is increased sufficiently. This tendency is more clarified from the plots of the particle displacements, u_x, u_y and u_z, shown in Fig. 3, which shows the relative displacements observed on the x axis between $x/a = 0$ and 1.0. This plots exhibit that the leading-displacement component u_x is indeed quite uniform along the x axis, while the component u_y is negligibly zero as that of the SH wave in a uniform plate of the thickness b is so, but that the u_z component is also observed significantly at around the stress-free boundary at the side surfaces. From further careful investigation on these results and also the displacement-vector plots (which are omitted here and will be presented at the talk), the modal behavior of the flexural mode seems to quite resemble to that of the Lamb wave on the elastic plate with the thickness a ($b \to \infty$). So, we have replotted Fig. 2 by replacing a normalization constant b in both axes with a as shown in Fig. 4. In this case, the dispersion curves of the flexural mode for the different a/b values fall approximately on one curve shown broken in Fig. 4, while the dispersion curve for the anti-symmetric Lamb wave on the elastic plate with the thickness a ($b \to \infty$) is shown by the thick-solid curve. Thus, both curves seem to be almost coincided as long as the frequency range shown there, and we can conclude that the modal-field behavior of the fundamental flexural mode on the acoustic rectangular waveguides is understood from that of the Lamb wave mentioned above. This result is just our new insight for the item (1) mentioned in Introduction.

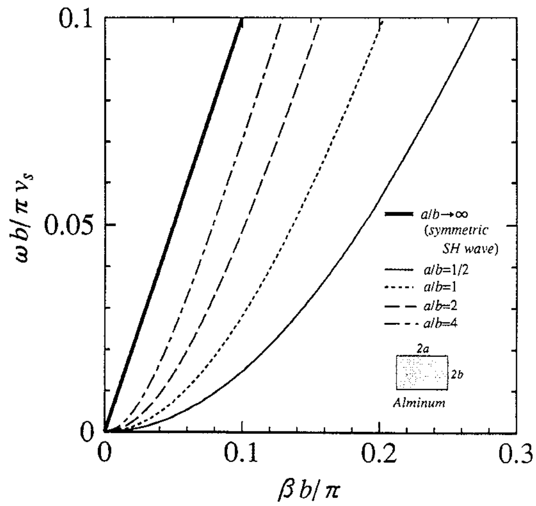


Fig. 2. Dispersion curves of the normalized frequency $\omega b/\pi v_s$ versus the normalized phase constant $\beta b/\pi$, for different a/b values.

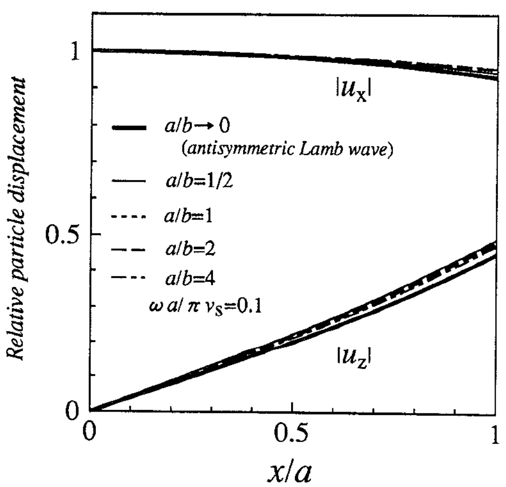


Fig. 3. Intensity distributions of the particle displacements u_x and u_z along the x axis for $a/b = 1.0$ and $\omega a/\pi v_s = 0.1$. Both results agree well with the distributions of the particle displacement of the Lamb wave when $b \to \infty$.

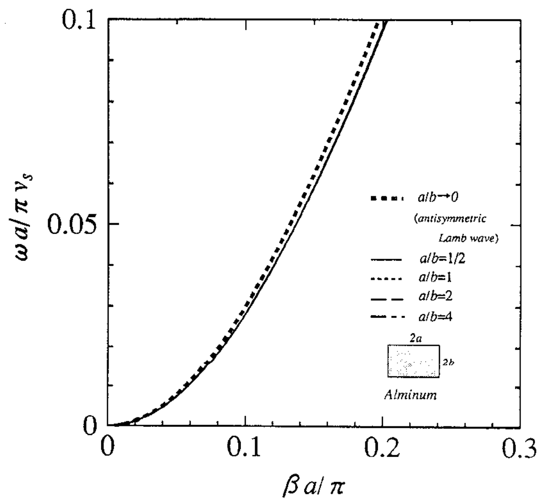


Fig. 4. Dispersion curves of the normalized frequency $\omega a/\pi v_s$ versus the normalized phase constant $\beta a/\pi$, for different a/b values.

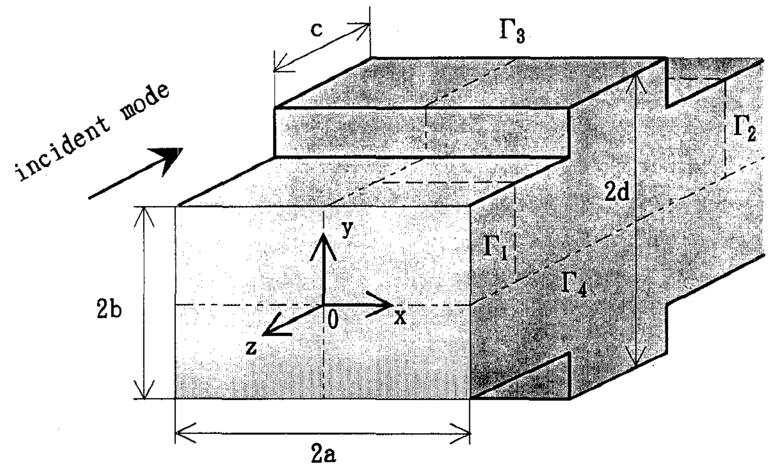
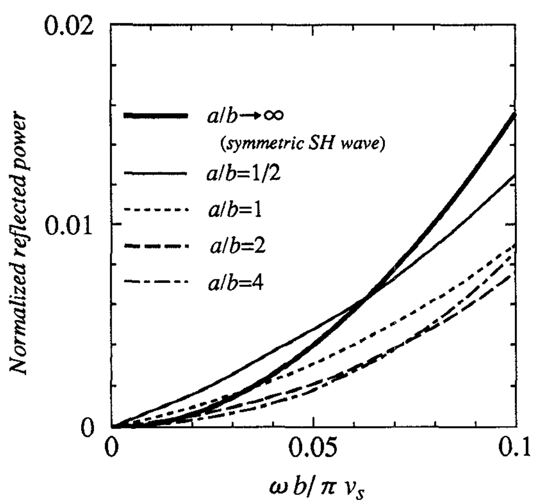


Fig. 5. Sketch of the double-step discontinuity on the acoustic rectangular waveguide. We assume that the fundamental mode impinges far from the front side.

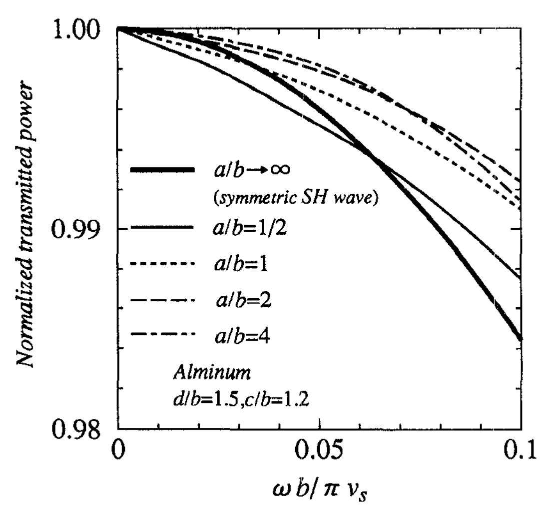
3. CHARACTERISTICS OF THE DOUBLE-STEP DISCONTINUITIES

We next discuss the discontinuity problem on the acoustic rectangular waveguide on the basis of the knowledge obtained in the previous section. Fig. 5 shows the sketch of the double-step discontinuity, for which the fundamental flexural mode impinges far from the front end. The accurate FEM-calculation results for the reflection and transmission characteristics are shown in Fig. 6(a) and Fig. 6(b), respectively, for the different a/b values. The other dimensional parameters are shown in the figures. The thick-solid curve indicates the results for the double-step discontinuity in uniform plates in the x direction ($a \to \infty$ in Fig. 5), but assuming the SH-wave incidence. Thus, such a simple 2D-discontinuity model is completely ineffective to calculate the reflection and transmission coefficients of the 3D discontinuity for the flexural mode. This result is just our solution for the item (2) mentioned in Introduction. Then, we have a question here: Should we always use a full-numerical method like FEM to analyze 3D-discontinuity problems?

As mentioned previously, however, the modal-field behavior of the fundamental flexural mode on the acoustic rectangular waveguides is understood from that of the Lamb wave existing on the elastic plate with the thickness a ($b \to \infty$). Since the condition of $b \to \infty$ is equalized with the symmetric plane placed parallel to the xz plane at any y position, we may replace both the top and bottom stress-free surfaces in each waveguide with the symmetric planes. As a result, the problem of Fig. 5 with the fundamental flexural-mode incidence is approximated by a 2D-discontinuity problem of the Lamb-wave incidence mentioned above. The results obtained by such an approximation are shown by the dot-broken curves in



(a)



(b)

Fig. 6. The reflection (a) and the transmission (b) coefficients versus the normalized frequency $\omega b/\pi v_s$.

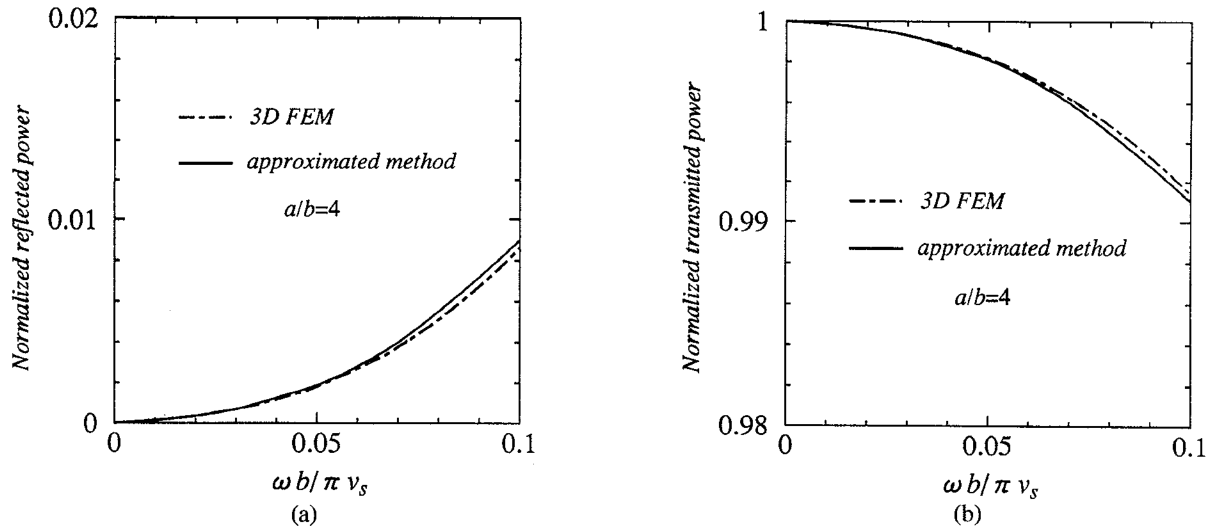


Fig. 7. Comparison of the reflection (a) and the transmission (b) coefficients between 3D FEM and the approximated 2D-discontinuity model.

Fig. 7, and actually show a good agreement with the accurate FEM results. Such an approximated 2D-discontinuity model can be, of course, solved by not the numerical method like FEM, but the analytical method, so that the calculation process is surprisingly simplified. For such an analytical approach, we can also apply the well-established microwave-circuit analysis and the equivalent network approach. These will be presented at the talk. As a result, we don't know, to our knowledge, such a successful investigation mentioned here on the discontinuity problems of the 3D acoustic rectangular waveguides, and, in this sense, the results and discussions are quite new, and this is just our solution to the item (3) mentioned in Introduction.

4. CONCLUSION

Since there is no systematic investigation on the modal-field behaviors, we have first tried to understand systematically the modal behavior, especially for the fundamental flexural mode, in the way similar to the electromagnetic-field theory, and we have explained it only by that of the Lamb wave, at least, for the structures discussed here and in the frequency range. Then these results have been applied to develop a simple 2D-structural model for solving an actual 3D discontinuity. Further detail of this approach and application to other acoustic-device analysis will be presented at the talk.

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